Quiz 2, Linear

Dr. Adam Graham-Squire, Fall 2017

5.5 min

$\Rightarrow$ 20 min in class

Name: Key

1. (4 points) Is the vector $\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$?

   Show your work or explain your reasoning.

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] ✓$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] ✓✓$$

has a free variable and is consistent, so answer is $\boxed{\text{yes!}}$ ~~There is~~ The vector $\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ is a linear combination of the other three. ✓

2. (3 points) Let $\mathbf{u} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$ for all h and k.

$$\left[\begin{array}{cc|c} 5 & 5 & h \\ -1 & 1 & k \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & \frac{h}{5} \\ 0 & 2 & k+\frac{h}{5} \end{array}\right]$$

✓

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & \frac{h}{5} \\ 0 & 1 & \frac{1}{2}\left(k+\frac{h}{5}\right) \end{array}\right]$$ ✓

This will <u>always</u> be consistent for any values of h, k b/c it has two pivots. So for any h, k, $\begin{bmatrix} h \\ k \end{bmatrix}$ is in the span. ✓

3. (3 points) Let A be a 4×3 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^4$.

$$A = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

If you had $Ax = b$, it would look like

$$\left[\begin{array}{ccc|c} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right]$$

when you row reduce, you can get at most 3 pivots in A, so you could get

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & K \end{array}\right]$$

There will be some ~~values of~~ vectors $\vec{b}$ such that this $K \neq 0$, wh... ...consistent.